

SU₂ × SU₂ Spin Flavor Subquark Model of Family Structure

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A simple spin-flavor model for the substructure of the 6 known quarks is proposed which is able to predict the correct Dirac magnetic moments if we interpret the quark masses as constituent masses. The addition of other degrees of freedom is proposed to facilitate the prediction of the correct mixing angles.

1. Introduction

To give an explanation for the existence of the quark mass matrix is one of the most pressing and deeply understood problems of present day particle theory. The fundamental problem is concerned with the fact that the weak eigenstates are not in a representation where the strong or mass eigenstates are diagonal. In other words, nature seems to mix the mass eigenstates that go into the Weinberg-Salam doublets. Fritzsche had attempted to derive relations between the mixing angles and the mass eigenvalues at the time when little was known about the quarks b and t [1]. The idea that heavy quark masses are driving terms in the mass matrix and the lighter masses are due to nearest neighbor mixing finds justification within left-right symmetric models and grand unified models [2, 3]. If we observe the elements in the K.M. mixing matrix according to Kleinknecht and Renk [4] we have

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

where

$$V = \begin{pmatrix} 0.9723 - 0.9737, & 0.228 - 0.234, & 0.000 - 0.004 \\ 0.228 - 0.234, & 0.974 - 0.9727, & 0.039 - 0.501 \\ 0.005 - 0.015, & 0.038 - 0.050, & 0.9987 - 0.9993 \end{pmatrix}$$

This matrix indicates that u, d mix with c, s on the average a power of 10 greater than t, b mix with c, s.

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The pattern of mixing also suggests an approximate orthogonality of the families with respect to charged currents while we know that perfect orthogonality is enjoyed by u – c, u – t, t – c along with d – s, d – b, s – b. In other words nature abhors flavor changing neutral currents. Of course the problem of CP violating phases also has to be explained and to date aside from mechanisms contrived using various Higgs fields a microscopic origin of CP violation has not been suggested.

Visnjic Triantafillou has developed a model of radial excitations wherein the c, t are S states of a radial potential with a bosonic preon moving about a fermion [5]. His argument is that

$$\psi_u(\mathbf{r}), \psi_c(\mathbf{r}), \psi_t(\mathbf{r})$$

are orthogonal to each other along with

$$\psi_d(\mathbf{r}), \psi_s(\mathbf{r}), \psi_b(\mathbf{r}).$$

However, because the parameters in the potential are different for the u type quarks and d type quarks the following overlap integrals are unequal to 0,

$$\begin{aligned} \int \psi_u^*(\mathbf{r}) \psi_b(\mathbf{r}) d^3x &\neq 0, \\ \int \psi_c^*(\mathbf{r}) \psi_d(\mathbf{r}) d^3x &\neq 0, \\ \int \psi_u^*(\mathbf{r}) \psi_s(\mathbf{r}) d^3x &\neq 0, \\ \int \psi_c^*(\mathbf{r}) \psi_b(\mathbf{r}) d^3x &\neq 0, \\ \int \psi_t^*(\mathbf{r}) \psi_d(\mathbf{r}) d^3x &\neq 0, \\ \int \psi_t^*(\mathbf{r}) \psi_s(\mathbf{r}) d^3x &\neq 0, \end{aligned} \quad (1.1)$$

and thus give rise to mixing angles. This picture strongly suggests a model for quarks built upon radial excitations multiplied by a wave function which contains the other subquark quantum numbers of spin, flavor, color, hypercolor, etc. In what

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follows, we develop a model originally suggested by Wetterich [6], namely that the generation structure emerges from 3 subquarks that have spin and isospin. Each combination of a spin-function and a isospin-function can lead to a possible quark-like particle. This model also somewhat resembles the model of Gonzalez-Mestres [7], wherein a weak isospin quantum number imposed on the preons is used to reproduce the spectrum of leptons and quarks.

2. Spin-Flavor Composites

If we choose to construct spin-flavor functions that represent quarks built out of constituent subquarks the symmetry of the resulting wave function is an important consideration to take into account. If we choose 3 subquarks we might have subquark 1 and 2 correlated in flavor space while subquarks 2 and 3 might be correlated in spin space. Thus

$$\Psi_{(1 \times 2)+3}^S \times \Psi_{1+(2 \times 3)}^F \neq \Psi_{(1 \times 2)+3}^S \times \Psi_{(1 \times 2)+3},$$

where S and F represent spin and flavor, respectively. This type of construction leads to a doubling of the possible number of quark-like states and also enforces the success of the model in predicting the correct magnetic moment values.

Let us consider the following assignment for subquark u, d (u, d label subquark, U, C, T, D label quark):

$$\Psi^F = \begin{pmatrix} u \\ d \end{pmatrix}, \quad \text{with charge matrix} \quad Q = \begin{pmatrix} \frac{5}{9} & 0 \\ 0 & -\frac{4}{9} \end{pmatrix}.$$

The conventional U, C, T quarks can be represented by

$$\Psi_{U,C,T} \sim (u \ u \ d) \times (\text{spin function}),$$

$$\Psi_{D,S,B} \sim (u \ d \ d) \times (\text{spin function}).$$

Table 1 lists the possibilities existing for spin $\frac{1}{2}$ composite states made of 3 subquarks including the different correlated spin-flavor functions mentioned above.

From the list of spin-flavor functions in Table 1 we now construct the magnetic moments of the various functions using

$$M_z = \sum_{i=1}^3 u_i \left(\frac{Q_i}{e} \right) S_{z,i} \quad (2.1)$$

in units $\hbar = c = 1$. In (2.1) $\mu_0 = \text{constant}$, $Q_i = \text{charge of subquark}$, $S_z = z \text{ component of spin}$, and we have assumed μ_0 to be the same for both u and d like subquarks. Calculating the value of

$$\langle \Psi | M | \Psi \rangle$$

for the states in Table 1 we obtain the magnetic moments listed in Table 2.

To construct observable quark wave functions we will choose linear combinations of the wave functions listed in Table 1 with the intent of reproducing the magnetic moments of the quarks using constituent quark masses. For the masses we use

$$m_U \cong m_D \cong 330 \text{ MeV},$$

$$m_C \cong 1350 \text{ MeV},$$

$$m_S \cong 500 \text{ MeV},$$

$$m_B \cong 5300 \text{ MeV} \quad \text{with } m_T \text{ uncertain [8].}$$

The Dirac magnetic moment is

$$u_z = \frac{e_q}{2m_q} (2) S_z \quad (2.2)$$

with

$$\frac{u_{q_1}}{u_{q_2}} = \frac{e_{q_1} m_{q_2}}{e_{q_2} m_{q_1}}.$$

To motivate the choice of composite quark functions in Table 1 we reason as follows: The Dirac magnetic moment given by (2.2) suggests that we use either Ψ_1 , Ψ_4 or Ψ_6 for the U quark since they have the largest Dirac moments from (2.1). The choice of Ψ_1 or Ψ_4 , however, makes it very difficult to choose the other functions for D, S, C, B, T that are consistent with (2.2) for the following reasons: The value u_0 in Table 2 has to be associated with the effective mass of the preons that are responsible for the individual preon Dirac magnetic moments. The effective preon mass is different for preons in the U type quarks as opposed to D type quarks. To a first approximation we assume that a quark is a charged sphere with effective electromagnetic energy Q^2/R and effective binding potential of one preon relative to the other to as being due to the very complicated hypercolor dynamics and given by the relation $V(R) = C_1 + C_2 R$, where we have written just the confining part of the potential suggested by Grosser, Falkensteiner and Schoberl [9]. For equilibrium we have the condition

$$\frac{d}{dR} \left(\frac{Q^2}{R} + C_1 + C_2 R \right) = 0, \quad \text{or} \quad R \propto Q,$$

Table 1. Spin-flavor functions for quark-like composite particles.

| U-like quarks Ψ | D-like quarks Ψ' |
|---|--|
| $\Psi_1 = \frac{(u d - d u) u}{\sqrt{2}} \left(\frac{\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow}{\sqrt{2}} \right)$ | $\Psi'_1 = \frac{(u d - d u) d}{\sqrt{2}} (\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)$ |
| $\Psi_2 = \frac{(u d - d u) u}{\sqrt{2}} \left(\sqrt{\frac{2}{3}} (\uparrow\uparrow\downarrow) - \frac{1}{\sqrt{6}} (\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) \right)$ | $\Psi'_2 = \frac{(u d - d u) d}{\sqrt{2}} \left(\sqrt{\frac{2}{3}} (\uparrow\uparrow\downarrow) - \frac{1}{\sqrt{6}} (\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) \right)$ |
| $\Psi_3 = \left(\sqrt{\frac{2}{3}} (u u d) - \frac{1}{\sqrt{6}} (u d u + d u u) \right) \left(\frac{\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow}{\sqrt{2}} \right)$ | $\Psi'_3 = \left(\sqrt{\frac{2}{3}} (d d u) - \frac{1}{\sqrt{6}} (u d d + d u d) \right) \left(\frac{\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow}{\sqrt{2}} \right)$ |
| $\Psi_4 = \left(\sqrt{\frac{2}{3}} (u u d) - \frac{1}{\sqrt{6}} (u d u + d u u) \right) \cdot \left(\sqrt{\frac{2}{3}} (\uparrow\uparrow\downarrow) - \frac{1}{\sqrt{6}} (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow) \right)$ | $\Psi'_4 = \left(\sqrt{\frac{2}{3}} (d d u) - \frac{1}{\sqrt{6}} (u d d + d u d) \right) \cdot \left(\sqrt{\frac{2}{3}} (\uparrow\uparrow\downarrow) - \frac{1}{\sqrt{6}} (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow) \right)$ |
| $\Psi_5 = \frac{u (u d - d u)}{\sqrt{2}} \left(\frac{\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow}{\sqrt{2}} \right)$ | $\Psi'_5 = \frac{d (u d - d u)}{\sqrt{2}} \left(\frac{\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow}{\sqrt{2}} \right)$ |
| $\Psi_6 = \frac{d (u d - d u)}{\sqrt{2}} \left(\sqrt{\frac{2}{3}} (\uparrow\uparrow\downarrow) - \frac{1}{\sqrt{6}} (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow) \right)$ | $\Psi'_6 = \frac{d (u d - d u) \mathbf{R}}{\sqrt{2}} \left(\sqrt{\frac{2}{3}} (\uparrow\uparrow\downarrow) - \frac{1}{\sqrt{6}} (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow) \right)$ |
| $\Psi_7 = \left(\sqrt{\frac{2}{3}} d u u - \frac{1}{\sqrt{6}} (u u d + u d u) \right) \left(\frac{\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow}{\sqrt{2}} \right)$ | $\Psi'_7 = \left(\sqrt{\frac{2}{3}} (u d d) - \frac{1}{\sqrt{6}} (d u d + d d u) \right) \left(\frac{\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow}{\sqrt{2}} \right)$ |
| $\Psi_8 = \left(\sqrt{\frac{2}{3}} (d u u) - \frac{1}{\sqrt{6}} (u u d + u d u) \right) \cdot \left(\sqrt{\frac{2}{3}} \uparrow\uparrow\downarrow - \frac{1}{\sqrt{6}} (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow) \right)$ | $\Psi'_8 = \left(\sqrt{\frac{2}{3}} (u d d) - \frac{1}{\sqrt{6}} (d u d + d d u) \right) \cdot \left(\sqrt{\frac{2}{3}} \uparrow\uparrow\downarrow - \frac{1}{\sqrt{6}} (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow) \right)$ |

where Q is the total quark charge. Thus, the effective size of the quark is proportional to the charge. Since the individual preons are point-like with respect to their Dirac moment, we conclude that, since the hypergluon cloud is distributed over a greater volume for preons in a U type quark as opposed to a D type quark, the effective mass that goes into the Dirac moment is inversely proportional to R^2 , or $(m_p)_U \propto 1/R_U^2$ in the case of planer preons with the rest of the mass being distributed over the hypergluon cloud which is bigger for U type quarks as opposed to D like quarks. Since

$$(u_0)_U \propto \frac{1}{(m_p)_U}, \quad (u_0)_D \propto \frac{1}{(m_p)_D}$$

we have

$$\frac{(u_0)_U}{(u_0)_D} = \frac{(m_p)_D}{(m_p)_U} = \frac{R_D^2}{R_U^2} = \frac{Q_D^2}{Q_U^2} = 4.$$

The neglect of the short range term in the Grosser potential can be attributed to its intrinsic small coefficient along with the cancellations suggested to us by the approximate condition of chiral symmetry respected by the preons. This argument, in combination with the fact that it is impossible to choose wave functions for D, S, C, B, T to fit the Dirac magnetic moments given by (2.2) if Ψ_1 or Ψ_4 is used for U, suggests the choice $\Psi_U = \Psi_C$. To insure the approximate validity of the relation $\langle \Psi_U | \Gamma_+ | \Psi_D \rangle = 1$ in spin flavor space in accord with the K.M. matrix along with a correct prediction for the ratio of the U to D Dirac moments suggests $\Psi_D = \Psi'_6$. Using the values in Table 1 along with the conversion factor of 4 discussed above we have

$$\frac{M_U}{M_D} = \frac{1}{18} \left(\frac{-108}{15} \right) \cdot 4 = -2. \quad (2.3)$$

Table 2. Magnetic moments divided by u_0 for functions listed in Table 1.

| Wave functions | $\langle \Psi M \Psi \rangle \cdot \frac{1}{u_0}$ | Wave functions | $\langle \Psi M \Psi \rangle \cdot \frac{1}{u_0}$ |
|----------------|---|----------------|---|
| Ψ_1 | $\frac{10}{36}$ | Ψ'_1 | $-\frac{2}{9}$ |
| Ψ_2 | $\frac{1}{54}$ | Ψ'_2 | $\frac{1}{9}$ |
| Ψ_3 | $-\frac{1}{18}$ | Ψ'_3 | $\frac{12}{108}$ |
| Ψ_4 | $\frac{10}{36}$ | Ψ'_4 | $-\frac{2}{9}$ |
| Ψ_5 | $\frac{1}{72}$ | Ψ'_5 | $\frac{1}{30}$ |
| Ψ_6 | $\frac{1}{18}$ | Ψ'_6 | $-\frac{15}{108}$ |
| Ψ_7 | $-\frac{2}{27}$ | Ψ'_7 | $-\frac{41}{54}$ |
| Ψ_8 | $\frac{1}{36}$ | Ψ'_8 | $\frac{7}{36}$ |

For the C quark we are guided by two requirements, first we must insure the absence of F.C.N.C. so that $\langle \Psi_C | \Psi_U \rangle = 0$, secondly, we must reproduce the correct ratio of $\frac{M_C}{M_U}$; we are, therefore, led to the wave function $\Psi_C = \Psi_5$. Using the values in Table 2 we have $\frac{M_C}{M_U} = \frac{18}{72} = 0.25$, as compared to 0.244 from the constituent masses. The match of 0.25 to 0.244 is very close but of course is dependent on our choice of the U constituent mass. To find a wave function for S we are guided by three criteria; first, we must insure the orthogonality of S and D in spin-flavor space; secondly, we must insure the correct prediction of the S magnetic moment using Table 2; thirdly, we must have that the inner product of Ψ_S with Ψ_C given by $\langle \Psi_C | \Gamma_+ | \Psi_S \rangle$ be close to 1 suggested by the K. M. Matrix.

To fulfill these requirements we have the following wave function representing S:

$$\Psi_S = \bar{a}_1 \Psi'_5 + \bar{a}_2 \Psi'_1, \quad (2.4)$$

with $\bar{a}_1 = 0.8$, $\bar{a}_2 = 0.6$. Then

$$\frac{M_C}{M_S} = \frac{4 \left(\frac{1}{72} \right)}{\frac{1}{36} (\cdot 8)^2 - (\cdot 6)^2 \left(\frac{2}{9} \right)} = -0.86,$$

after neglecting any overlap between Ψ'_5 and Ψ'_6 when inserting in (2.1) and the expression for M

given by $\langle \Psi | M | \Psi \rangle$. Using the constituent masses we have

$$\frac{M_C}{M_S} = -2 \left(\frac{500}{1350} \right) = -0.75. \quad (2.5)$$

Thus the calculation using the wave function (2.4) leads to a prediction which differs with the known value of the $\frac{M_C}{M_S}$ ratio to within 10%. We know, however, that the magnetic moment of the S quark is only known to within 10% because of the difficulty in measuring it [10]. For consistency we calculate

$$\frac{M_S}{M_D} = \frac{\frac{1}{32} (\cdot 8)^2 - (\cdot 6)^2 \left(\frac{2}{9} \right)}{-\frac{12}{108}} = 0.558, \quad (2.6)$$

using

$$\frac{M'_6}{u_0} \cong -\frac{M'_3}{u_0} = \frac{-12}{108},$$

and from the constituent quark masses

$$\frac{M_S}{M_D} = \frac{330}{500} = 0.66,$$

which is close to the result in (2.6). For the T quark we use the fact that Ψ_2 and Ψ_3 have equal and opposite magnetic moments, which suggests a linear combination to represent the T quark, which has a small Dirac moment. For T we are guided once again by the lack of F.C.N.C. and the smallness of the Dirac moment; these criteria suggest the following linear combination for T:

$$\Psi_T = \bar{a}_1 \Psi_6 + \bar{a}_2 \Psi_3. \quad (2.7)$$

We see from (2.7) that $\langle \Psi_T | \Psi_C \rangle = 0$. Now

$$\frac{M_T}{M_C} = \frac{(\bar{a}_1^2 - \bar{a}_2^2) \frac{1}{18}}{\frac{1}{72}} = 4(\bar{a}_1^2 - \bar{a}_2^2).$$

If we provisionally choose $m_T \approx 30$ GeV, we have

$$\frac{M_T}{M_C} = 4(\bar{a}_1^2 - \bar{a}_2^2) = \frac{m_C}{m_T} = \frac{1.35}{30} = \frac{1}{22}.$$

We find

$$\bar{a}_1^2 - \bar{a}_2^2 = \frac{1}{88} \quad \text{or} \quad \bar{a}_1 = \sqrt{\frac{89}{176}}, \quad \bar{a}_2 = \sqrt{\frac{97}{176}}.$$

We have also used the normalization condition $\bar{a}_1^2 + \bar{a}_2^2 = 1$ to calculate \bar{a}_1, \bar{a}_2 . For the bottom quark we are guided by the approximate conditions suggested by the K.M. matrix $\langle \Psi_T | \Gamma_+ | \Psi_B \rangle = 1$, along with the absence of F.C.N.C. symbolized by

$\langle \Psi_B | \Psi_S \rangle = 0$. We thus write

$$\Psi_B = \bar{a}_1 \Psi'_6 + \bar{a}_2 \Psi'_3 \quad (2.8)$$

with

$$\frac{M'_6}{u_0} = \frac{-15}{108} \cong \frac{-12}{108}, \quad \frac{M'_3}{u_0} = \frac{12}{108}.$$

For M_T/M_B we have

$$\frac{M_T}{M_B} = \frac{4 \left(\frac{1}{18} \right) \left(\frac{1}{88} \right)}{\frac{-12}{108} (\bar{a}_1^2 - \bar{a}_2^2)} = -2 \left(\frac{5.3}{30} \right),$$

yielding

$$\bar{a}_1 = \sqrt{\frac{16}{30}}, \quad \bar{a}_2 = \sqrt{\frac{14}{30}}.$$

The final consistency check is

$$\frac{M_B}{M_S} = \frac{\frac{12}{108} \left(\frac{1}{15} \right)}{\frac{-1}{36} (\cdot 8)^2 + (\cdot 6)^2 \left(\frac{2}{9} \right)} = 0.11. \quad (2.9)$$

From the constituent masses we have

$$\frac{M_B}{M_S} = \frac{m_S}{m_B} = \frac{500}{5300} = 0.094, \quad (2.10)$$

which indicates a rough agreement of the Dirac values and the value calculated by our subquark scheme. The interesting point is that using a value of $M_T = 30$ GeV we are led back to a reasonable value of M_B/M_S . The results above are summarized in Table 3.

We observe we cannot calculate the mixing angles from for instance

$$\langle \Psi_U | \Gamma^+ | \Psi_S \rangle.$$

Since we must also know the spatial part and perhaps the color and hypercolor part of the wave function to evaluate the inner product of the states. In this regard, we also observe that Ψ_S also contains a small piece due to Ψ'_6 , since $\langle \Psi'_3 | \Gamma^- | \Psi'_6 \rangle = \langle \Psi'_1 | \Gamma^- | \Psi'_6 \rangle = 0$ but $\langle \Psi'_6 | \Gamma^- | \Psi'_6 \rangle = 1$ in order to generate the spin flavor component of the Cabbibo angle for U, S. A more precise function for Ψ_S would be

$$\Psi_S = b_1 \Psi'_3 + b_2 \Psi'_1 + b_3 \Psi'_6.$$

Table 3. Choice of quark wave-functions that reproduce the correct Dirac magnetic moments.

| Quark | Wave function |
|-------|---|
| U | Ψ_6 |
| D | Ψ'_6 |
| C | Ψ_6 |
| S | $0.8 \Psi'_3 + 0.6 \Psi'_1$ |
| T | $\bar{a}_1 \Psi + \bar{a}_2 \Psi_3, \quad \bar{a}_1^2 - \bar{a}_2^2 = \frac{1}{88}$ |
| B | $\bar{a}_1 \Psi'_6 + \bar{a}_2 \Psi'_3, \quad \bar{a}_1^2 - \bar{a}_2^2 = \frac{1}{15}$ |

The functions in Table 3 do predict the absence of F.C.N.C. since

$$0 = \langle \Psi_U / \Psi_C \rangle = \langle \Psi_C / \Psi_T \rangle,$$

along with

$$\langle \Psi_D / \Psi_S \rangle = \langle \Psi_S / \Psi_B \rangle = 0.$$

To give an accurate account of mixing angles we would need to know the spatial part of the wave functions. If we conjecture that the dominant part of the mixing angles come from the spin flavor functions we have

$$V_{UD} = 1, \quad V_{CS} = 0.8, \quad V_{TB} = 1, \quad V_{UB} \cong 0.71.$$

The last figure violates unitary of the mixing matrix and suggests that radial wave functions and overlap integrals are necessary to calculate the mixing angles. It is probably suggestive of the fact that the subquarks are strongly excited radially.

In conclusion, it is nevertheless encouraging that the correct Dirac magnetic moment can be fitted so easily by the wave functions of Table 3 without contrived mechanisms to generate the wave functions.

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